QUANTITATIVE PERSPECTIVES

AN IMPLIED PREPAYMENT MODEL FOR MBS

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ANDREW DAVIDSON & CO., INC.

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Introduction

In this article we describe a market-implied prepayment model for residential mortgage-backed securities. The methodology takes, as given, the market prices of securities together with a yield curve environment and produces a small set of parameters which describe a prepayment model.

This prepayment model can, then, be used on a broader portfolio to obtain market-implied valuation and risk measures. In addition to having potential hedging applications, this technology is a useful gauge of market sentiment over time and can be used to guide the evaluation of a prepayment model based on historical prepayment data.

A Two-Bond Example

We begin by motivating the development of an implied prepayment model with a two-bond example. Given two FNMA 30 year bonds with net coupons of 6.5 and 8.0 and the current yield curve environment, an option-adjusted spread (OAS) is usually calculated by generating a set of interest rate paths consistent with the yield curve, generating prepayments using the longer end of the simulated yield curves, and using the simulated short rates plus a spread to discount these cash flows. The OAS is that single number which, when added as a spread to all the rate paths, matches the average net present value of cash flows from the paths to the market price. In this analysis the prepayment model and market prices are given and we solve for OAS.

Instead of solving for OAS, we could try to speed up and slow down the prepayment model being used by scaling its output SMMs by a constant factor and try to match the OASs of the two bonds; since the OAS is the excess spread paid for the bond by the market after taking prepayment risk into account, if other characteristics of the bonds are similar, it would be reasonable to expect their OASs to be equal. Table 1 applies this idea to a FN6.0 and FN8.0 using the Andrew Davidson & Co., Inc. prepayment model (ADP) with an October 27th, 2000 treasury yield curve.

Table 1

FN6.0 and FN8.0 OASs using three ADP speeds and 10/27/00 data

	Price	OAS at 100% ADP
FN6	94.34	118
FN8	101.56	132
		OAS at 80% ADP
FN6		110
FN8		140
		OAS at 120% ADP
FN6		126
FN8		125

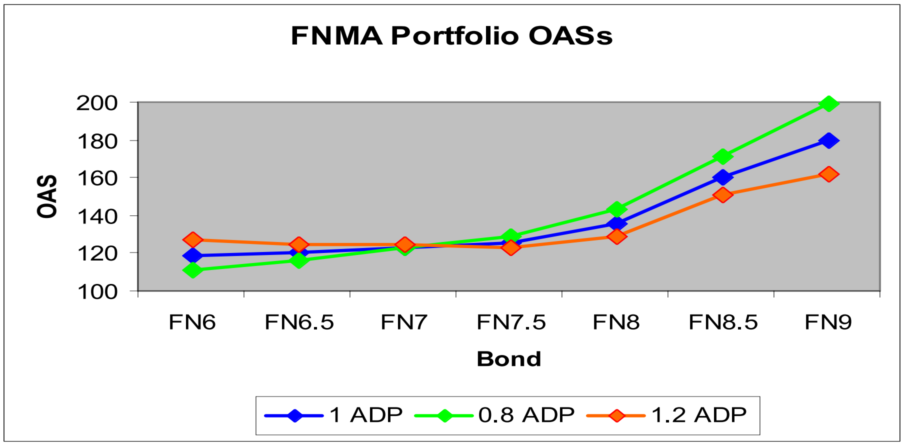
Treasury Curve in Months	Yield
3	6.335
6	6.372
12	6.159
24	5.930
60	5.785
120	5.713
360	5.741
Volatility %	17
Reversion %	2
No. of Paths	50

The first analysis shows the OAS of the two bonds using an unadjusted ADP model. When we decrease the prepayment speed by applying 80% of ADP, the timing of principal is extended for both bonds. For the discount this means a lower spread is required to match the same price, and for the premium, a higher spread. In the final run, we apply 120% of ADP; this shortens principal timing, increasing net present value for the discount and decreasing it for the premium. Hence, this increases the OAS of the discount and decreases the OAS of the premium. The values of the OAS match almost exactly at 120% of ADP.

In Figure 1 we plot the OASs of FN6-FN9 bonds under the three scalings used earlier with the same yield curve and their market prices from that date.

Figure 1

A FNMA Portfolio with 10/27/00 Market Prices

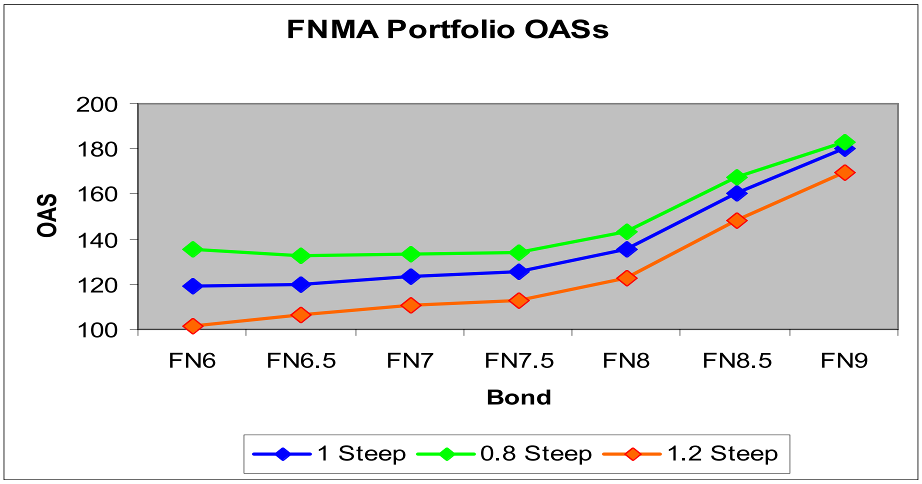


With more than two bonds, it is not always possible to match all the OASs using a simple scaling. While it is clear that the FN6.5 through FN8 bonds have almost the same OAS using 120% of ADP, some more flexibility is required to bring the OASs of the other bonds closer to some common OAS.

The Andrew Davidson & Co., Inc. prepayment model has additional tuning features which give us this flexibility. The *steepness* tuning parameter changes the shape of the refinancing incentive curve.[1] This leads to faster prepayments on premiums and slower speeds on discounts in one direction and the reverse in the other direction. Figure 2 displays the same OAS profile curves under 20% changes in this parameter.

Figure 2

The Effect of the Steep parameter



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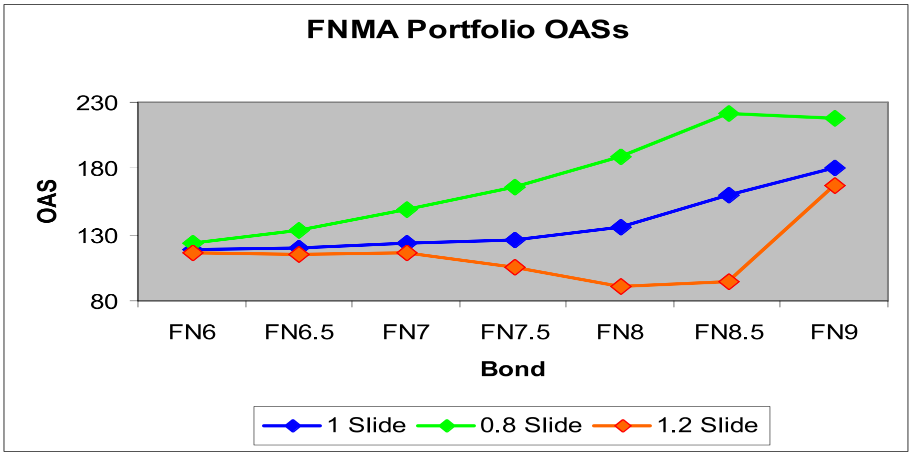
[1] With the ratio between the gross rate and market rates on the x-axis and the refinancing incentive, which can be thought of as the CPR due to refinancing, on the y-axis.

In contrast to Figure 1, where increasing the scale parameter seems to rotate the OAS profile clockwise while keeping the center relatively constant, the OAS profile moves up and down; in addition, the slopes of different parts of the curve can be changed by different amounts. This flexibility is useful when we try to equate the OASs of more than two bonds.

The *slide* parameter controls the ratio at which the refinancing incentive curve is "at the money." Decreasing this parameter changes a slight discount to a deeper discount while increasing it changes a discount to a premium. Figure 3 shows the effect on the OAS profile curve.

Figure 3

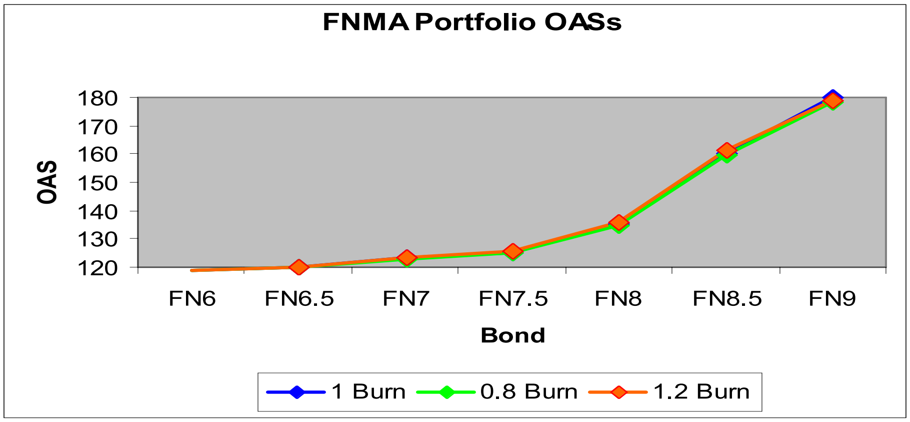
The Effect of the Slide parameter



The burnout tuning dial tends to have the least impact on valuation: on discounts it tends to have no impact at all, and on premiums, an impact that is small relative to the first three tuning dials. This can be seen in Figure 4.

Figure 4

The Effect of the Burnout parameter

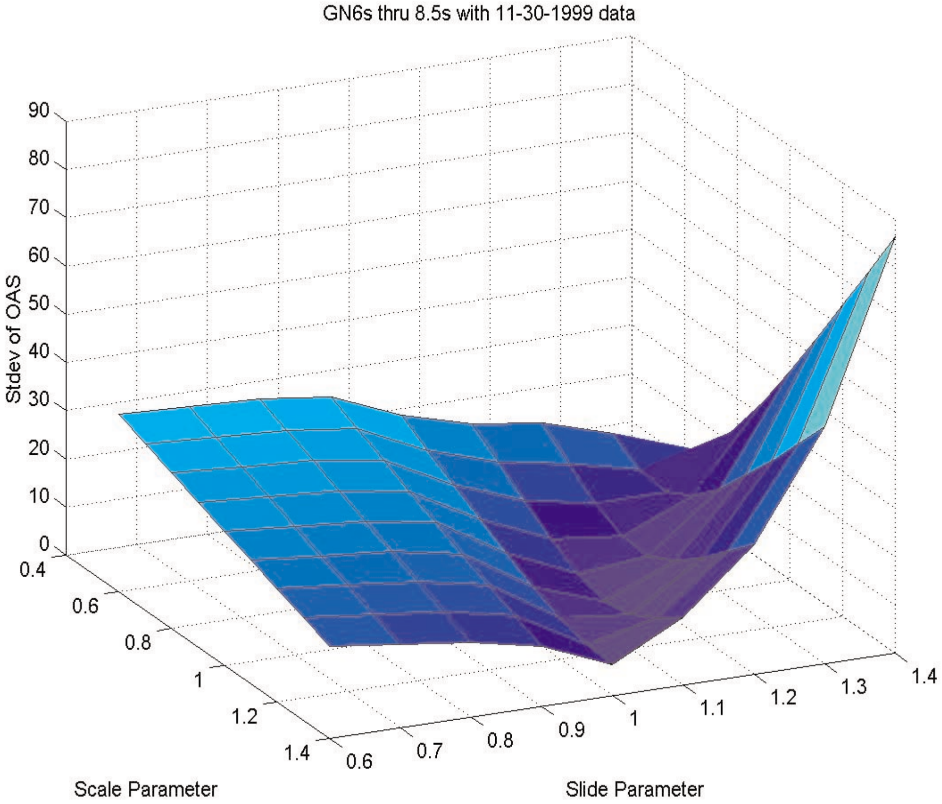


Because the bonds in the FNMA portfolio have the same credit quality, we would expect their OASs to be the same under the market-implied prepayment model.[2] This is because the OAS is the total spread minus the option cost; differences in prepayment risk have already been accounted for in the option cost.

We can use the standard deviation of the OASs of the portfolio of bonds as a measure of the dispersion of the OASs. Using this definition, the implied prepayment model is, then, the prepayment model (defined implicitly through the tuning parameters) which minimizes this measure of dispersion.[3] Figure 5 shows a 3-dimensional representation of a surface on which we would search for the minimum if we were using only two tuning parameters.

Figure 5

A Surface Using Two Parameters



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[2] Assuming there is no liquidity premium for some of the coupons. We will revisit this assumption later.

[3] It is also possible to use a target vector of OAS.

While the general optimization would be in 5 dimensions (with 4 tuning parameters), this simpler case makes several things clear. First, it is possible that the boundary of the acceptable-tuning parameter space will be hit. There is no guarantee that the value of OASs for the portfolio at such a point would make any sense. In addition, there appear to be several regions where the standard deviation is fairly low; to choose between such regions, it would be useful to have some sense of a preferred region and some ability to distinguish between nonsensical OAS values and reasonable OASs built into the model. Finally, to obtain a minimum to within 5-10 bp, it is clear that a simple grid search would have to search points no further apart than 0.1 in the space of tuning parameters.

Methodology and Software Implementation

A complete grid search of tuning parameters with a 0.1 spacing and each tuning parameter ranging between 0.5 and 1.5 would force us to perform over 14,000 portfolio OASs. This amounts to over an hour for each instrument (assuming that we are dealing with TBAs and not CMOs) in our portfolio. In addition, this method does not include any information about preferred regions or output OAS values that are acceptable.

For these reasons, we chose to implement the implied prepayment model using penalty functions and a state-of-the-art non-linear least squares optimization technique.[4] The penalty function essentially says that the market view should not severely disagree with a view based on historical prepayments. Thus, any movement away from the default tuning parameters of 1.0 must be earned by lowering the standard deviation of OAS, and the further away from the default values, the larger the burden. One way to specify this linearly would be that each 1% change in tuning must be earned with a 1 bp reduction in the standard deviation of OAS per security in the portfolio.

In addition, our analysis showed that seasoned, high-coupon loans had consistently higher OASs, and these could not be decreased within our framework. Generally, these loans tended to be illiquid and the accuracy of the price information on them was in question. Rather than completely

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[4] The algorithm is a subspace trust region method based on the inferior reflective Newton method.

discarding these higher coupons, we used weights on the different coupons. This allows us to place different amounts of emphasis on the information contained in the prices and OASs of different securities.[5]

The non-linear optimization was performed using the *lsqnonlin* function in the optimization toolbox in the MATLAB environment. This function is a general non-linear search method which can be used when information about the slope of the surface on which the optimization is being performed is not available in closed-form. The software implementation involves this function calling our MBSOAS and CMOOAS libraries via the *mexFunction* feature of MATLAB.

This methodology is currently available to anyone who has access to the MATLAB environment. In the future we plan to design a stand-alone routine which has these capabilities.

Some Results

Figure 6 displays October 27, 2000 results from the implied-prepayment model for GN30 and FN30 portfolios with coupons from 6 through 9.5.[6]

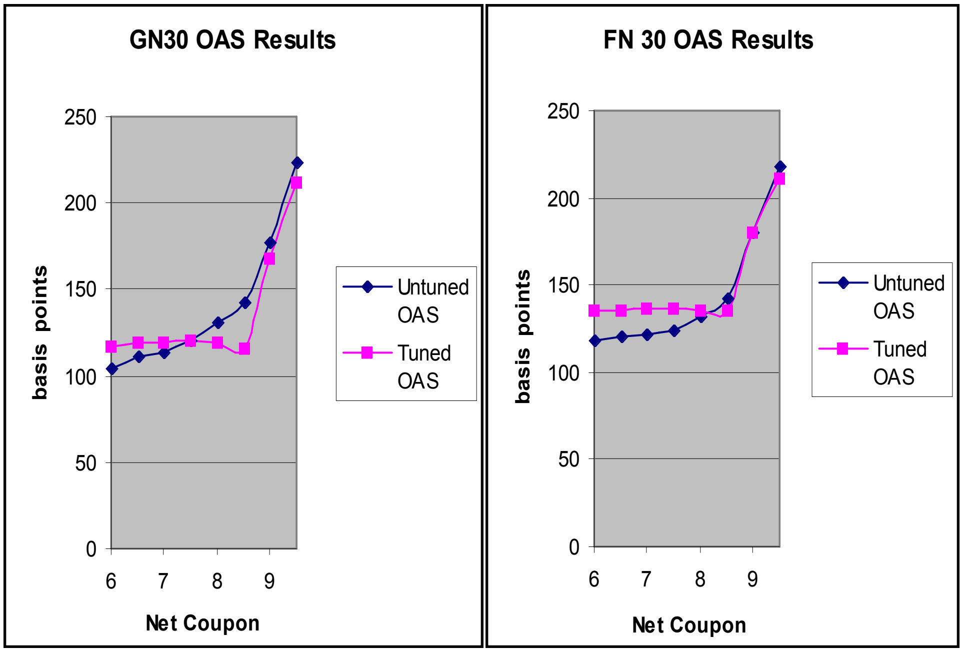

Figure 6
GN30 and FN30 OAS runs for 10/27/00

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[5] We can use weights that represent the amount of trading on those coupons which occurred over a certain period, or the proportion of outstanding loans the coupons represent, etc.

[6] The full set of WAC/WAM and volatility assumptions are available on our website at www.ad-co.com under Quantitative Analysis.

The actual OAS and tuning values are displayed in Table 2.

Table 2

GN30 and FN30 Before and After Tuning

	GN 30-Year	
Coupon	Untuned OAS	Tuned OAS
6	104	117
6.5	111	119
7	114	120
7.5	120	120
8	131	119
8.5	142	116
9	177	168
9.5	223	211
Scale	1	1.1358
Steep	1	0.9270
Slide	1	1.0798
Burn	1	1.0385

	FN 30-Year	
Coupon	Untuned OAS	Tuned OAS
6	118	136
6.5	120	135
7	121	137
7.5	124	136
8	132	136
8.5	142	136
9	180	180
9.5	218	211
Scale	1	1.1072
Steep	1	0.9002
Slide	1	1.0294
Burn	1	1.0052

First, if we look at the standard deviation before and after optimization for the whole portfolio, for the GN30s it decreases from 41 to 35bp and for FN30s from 36 to 29bp. However, we weighted the 9 and 9.5 coupons significantly less. If we look exclusively at the 6 through 8.5 coupons, for GN30s the standard deviation decreased from 14 to 2bp and for FN30s from 9 to less than 1 bp.

When the method weighed the higher coupons equally, it tended to lead to absurd results. There are several explanations for this discrepancy. First, the price data on the highest coupons is not as accurate as for the more liquid ones. Secondly, there may be a liquidity premium for the higher coupons. Finally, the market may be assigning a spread based on faster prepayments than the base prepayment model for those coupons under falling rates in a way that is not consistent with the structure of the model for the lower coupons.

For both collateral types, the market seems to favor a slightly less steep refi incentive curve than our base model combined with a speeding up overall. In addition, the values of slide indicate that securities are becoming "in the money" in the market model at slightly lower ratios than in our base model. This is shown in Table 3, where we compare the base case (static interest rate environment) WAL equivalent PSAs for the two collateral types under the untuned prepayment model and under the market-implied model.

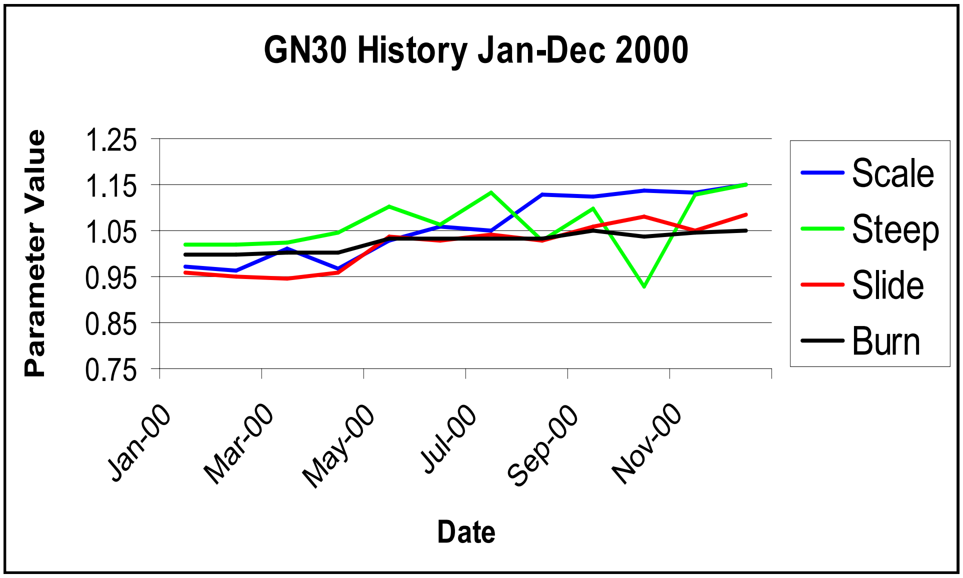
Table 3
Equivalent PSAs Before and After

Security	Untuned Base PSA	Tuned Base PSA
GN6	123	164
GN6.5	130	173
GN7	132	178
GN7.5	137	298
GN8	219	460
GN8.5	355	584
GN9	316	392
GN9.5	301	363

Security	Untuned Base PSA	Tuned Base PSA
FN6	135	174
FN6.5	139	181
FN7	144	189
FN7.5	173	277
FN8	310	435
FN8.5	471	642
FN9	375	419
FN9.5	380	418

Finally, we look at the series of solutions over time. Figure 7 shows the tuning parameters for the GN30 portfolio based on market close data from the last Friday of each month between January and December 2000.

Figure 7
Tuning Parameters for GN30s for Jan-Dec 2000



The scale and burn parameters seem to be the most stable over time for GN30s. Steep does seem to fluctuate over time, both above and below the steepness of the base model. However, scale and slide are consistently above 1, within a fairly small (5%) band.

It is important to note that the results of the implied prepayment model do not necessarily provide a forecasting model. Rather, the parameters specify a prepayment model which reflects the market's assessment of risk in much the same way that forward rates reflect the market's assessment of yield curve movements, or implied volatilities from caps and swaptions reflect the market's view of interest rate volatility. In all these cases, the implied curve or set of parameters is a composite view based on the often divergent views of different market participants.

Further Studies & Enhancements

We will continue to examine the relationship of the tuning parameters output by the implied prepayment model over time. In addition, we will begin to examine the stability of the implied prepayment model using the swap curve to forecast mortgage current coupon rather than the treasury curve.

While we used the dispersion of OASs within a portfolio to infer a prepayment model, this technology can be used with any other measures which can be fit to a reasonable target vector, or for which it makes sense to minimize some measure of distance. For example, we could solve for a prepayment model that produces a set of effective durations for our portfolio that matches a set of target durations.

A useful empirical test is to compare hedging performance using the base prepayment model against the implied prepayment model over some period of time. Finally, this technology allows us to compare across different prepayment models by stating one in terms of the other's tuning parameters. All of these topics will the be subject of further research at Andrew Davidson & Co., Inc.

Conclusion

The market-implied prepayment model estimation methodology takes, as given, a yield curve environment, volatility assumptions and the prices of MBS securities and outputs a set of tuning parameters which specify a prepayment model. In contrast to a prepayment model based on historical data, this implied prepayment model is one which minimizes the dispersion of OASs among the portfolio securities. The methodology can be used both to track market sentiment as the basis of an alternate hedging strategy and as a comparison between different views of prepayments and prepayment models. The general strategy can be used with measures other than OAS.

Eknath Belbase *is responsible for the research, development and testing of valuation/risk management tools and quantitative methodology for Andrew Davidson & Co., Inc. Eknath applies his mathematical and statistical experience to the development of cutting edge quantitative financial tools that are efficient, accurate and well-tested. He has worked on the MBS fixed-rate and ARM prepayment models, designed a lattice implementation of the Black-Karasinki interest rate process tailored to be extremely efficient on mortgage instruments, and researched LIBOR-Treasury and mortgage current coupon dynamics. He has worked on consulting projects involving structured products, credit risk and regulatory testing.*

Eknath holds a PhD in mathematics with specialization in probability, and an MS in Statistics, both from Cornell University. He received his BA in mathematics and computer science from Ohio Wesleyan University.

Eknath Belbase has also written:

A Lattice Implementation of the Black-Karasinski Rate Process	*June ‘00*
Fixed Rate Mortgage Prepayment Model	*March ‘00*

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