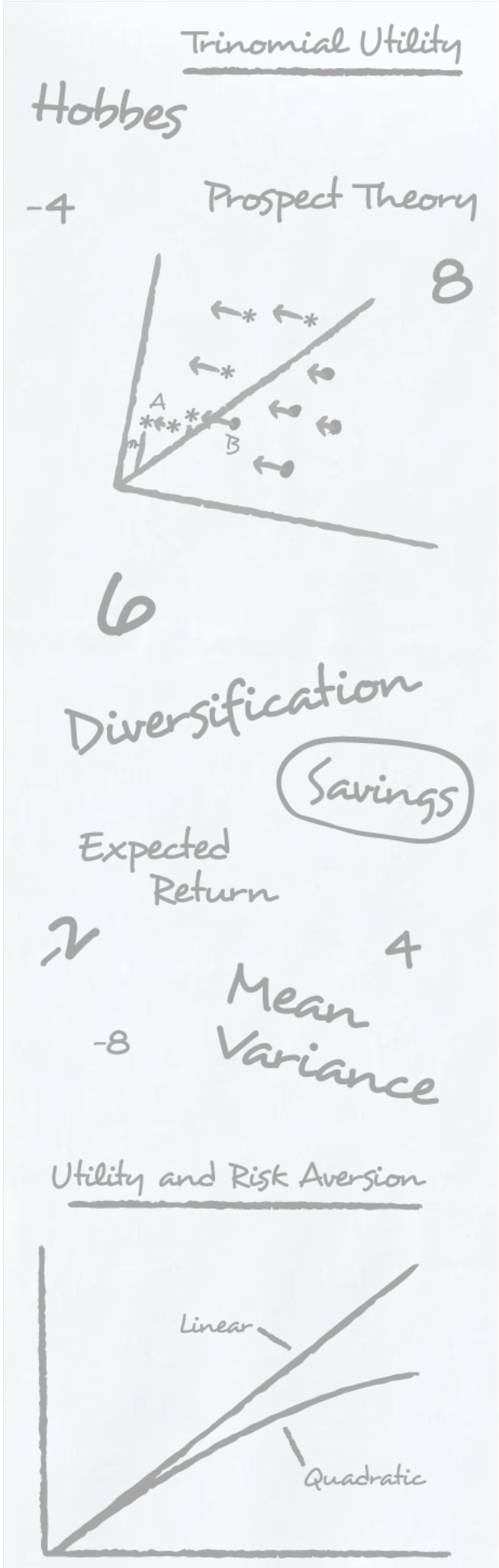


ANDREW DAVIDSON & CO

Household Utility and Financial Intermediation: A Parable

By Andrew Davidson

July 2020

Financial theories, such as the capital asset pricing model, are typically used to describe a fully functioning economy filled with a variety of assets along with access to financing and information, all at relatively low transaction costs. But let's consider a financial theory that begins in the state of nature: a theory of capital markets that starts with people in their natural financial state and then determines why they would establish financial intermediaries; a theory similar to that described by John Locke with respect to politics.

According to Locke in his *Second Treatise of Government*:

> *To understand political power correctly and derive it from its proper source, we must consider what state all men are naturally in. In this state, men are perfectly free to order their actions, and dispose of their possessions and themselves, in any way they like, without asking anyone's permission—subject only to limits set by the law of nature.*

So why do we give up that freedom and subject ourselves to governments? Because, Locke says,

> *In a state of nature where there is no authority to decide between contenders, and the only appeal is to heaven, every little difference is apt to end up in war; and that is one great reason for men to put themselves into society, and leave the state of nature. For where there is an authority, a power on earth from which relief can be had by appeal, the controversy is decided by that power and the state of war is blocked.*

But then why do we have financial intermediaries? A story may help.

In the financial state of nature

John Locke and Thomas Hobbes walk into a pub, called "The Whale." It operates on the border between societies with and without financial intermediaries. John is happy to be in a society that enforces property rights and even has established a currency as a means of exchange. (How else could they buy a beer?) But, so far there are no financial intermediaries in his society, so borrowing is not that easy and trading investments is very costly. Nevertheless, John is not sure he wants much more.

John Locke

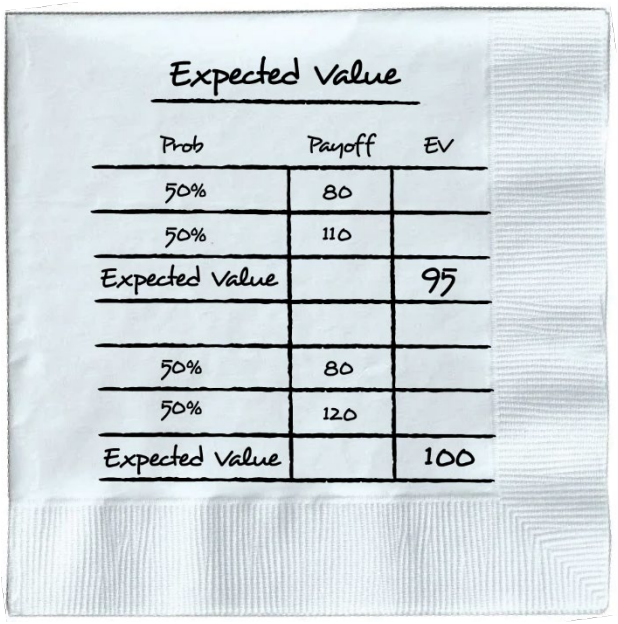
His friend and sometime sparring partner, Thomas buys the first round and pitches John with the idea for an investment. Without banks, Thomas is having a hard time getting a loan, and without stock exchanges and mutual funds,

Thomas needs to find one or at most a few investors who will commit to his venture until completion. In exchange for an investment in the venture now, Thomas tells John there will be future benefit. Thomas tells John that if he invests 100 gold coins now and the investment works out, he'll pay John 110 gold coins. If it doesn't work out, John should get back most, but not all, of his money.

Thomas Hobbes

John weighs the pros and cons. Ten percent seems like a pretty good return. The investment may allow John to make some more money, but John is not sure of the risks. Given his limited resources, John would have difficulty investing in multiple ventures. Once invested with Thomas or in another venture, he would need to stay committed to the venture until completion and if it fails, he will be out some money, which will certainly cut down on his beer consumption.

John wants to figure out if this is a good investment. Being more of a writer than a numbers guy, he consults with some local mathematicians and economists at the end of the bar. Fortunately, Nick and Daniel (Bernoulli 1738) are there. They suggest that at a minimum he needs to compute the expected value of the outcome. They suggest that John weight the payoff by its probability. They tell him that he may want more, but it's a start.

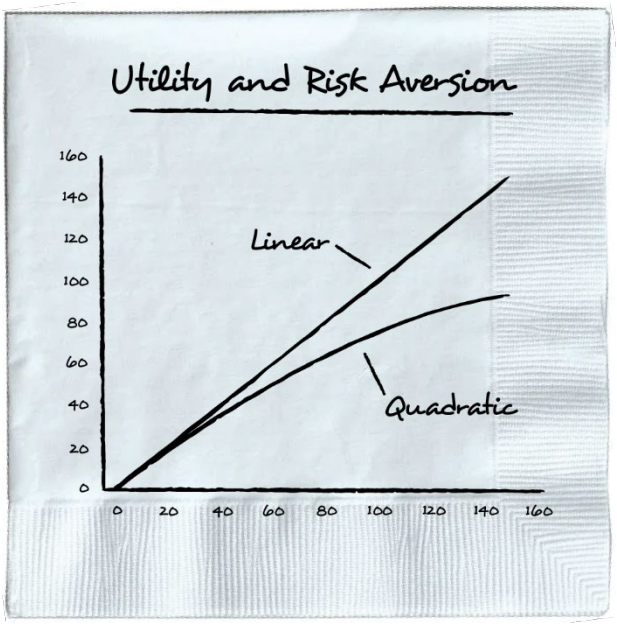


Napkin 1: Expected Value

John goes back to Thomas and asks about what could go wrong. Thomas says there is a 50% chance that he will only be able to give John 80 coins back if the venture doesn't work and he needs to shut down. The Bernoulli cousins write on what turns out to be the first of many napkins and show John that he needs to get at least 120 gold coins if the project succeeds to offset the potential loss of 20 gold coins 50% of the time. The Bernoulli cousins point out that while that the expected value method is a start, John might want to consider the risk. The cousins also talk about utility, logarithms, and other functions, leaving John confused and asking some more questions.

Meanwhile, Hobbes is now facing a tough choice. He didn't really want to pay out more than 110 gold coins as he thought 10% was a pretty good return to offer, but if he can't get John to invest, he may need to leave the state of nature and join a society with some financial intermediaries. He

knows this means giving up a lot of freedom. There may be taxes, disclosure requirements and other dreaded regulation. He decides to keep at it with John.

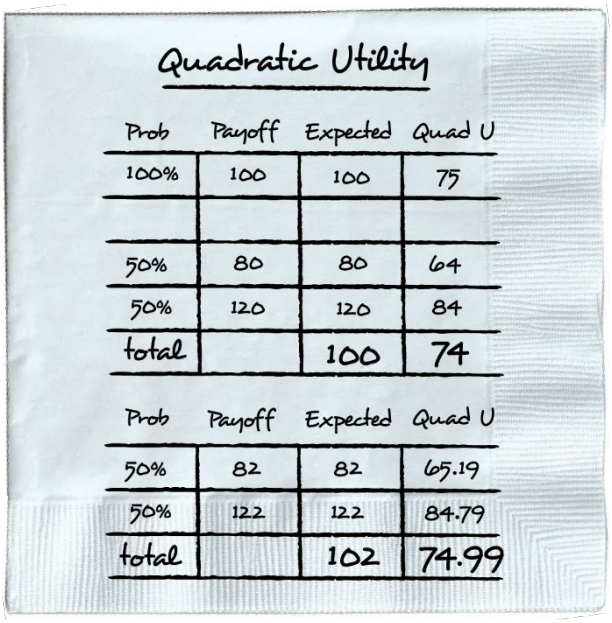


Napkin 2: Utility and Risk Aversion

John is still worried about the risk of the investment. John prefers a certain outcome to a range of possible outcomes: Why give up 100 gold coins just for an investment that gives you 100coins on average? Fortunately, John V and Oskar M (Von Neuman-Morgenstern 1944) have entered the pub. Locke buys them each a pint and asks if they can help him understand what it would take for him to invest in Thomas's venture. John and Oskar tell Locke that he needs to think about utility. "We have axioms and postulates to let you figure out how much you need to earn. The basic idea is that your utility curve, which is a measure of the value of gold coins to you, needs to have certain properties."

John and Oskar take out a napkin and draw some pictures for Locke. They explain that the straight line is expected value. It treats all proceeds the same. The curved line shows risk aversion. As the payoff increases, the utility of the payoff is still increasing, but at a decreasing rate. One simple example is quadratic utility.

John and Oskar help Locke figure out how much he needs to invest with Thomas. With quadratic utility, if Locke received a payoff of 100 it would have a utility of 75. A 50/50 combination of 80/120 would only have a utility of 74. Since that is less than the utility of just keeping his 100 coins, John should insist that Thomas increase his payoff. A payoff of 82/122 would have a utility equal to just keeping the 100, so that's the minimum Locke should accept.

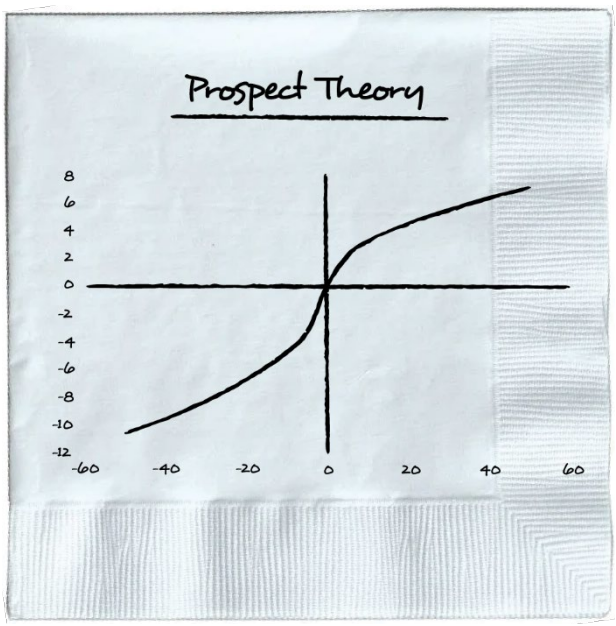


Napkin 3: Quadratic Utility

Thomas doesn't like the way this conversation is going. He now needs to shell out another two gold coins. Locke says this is starting to make sense: If I have more risk, I need more return.

John gets ready to invest with Thomas but spits out his beer and says, "It still doesn't feel right to me." Under your rules I should be willing to take some risk with my money even if there is just a small return. If there was an investment that either paid one gold coin or lost one gold coin, you tell me I would want to invest if you threw in an extra

farthing or two. Regardless of your theories, that just doesn't feel right to me. I just don't like the risk of losing even if is just one coin."

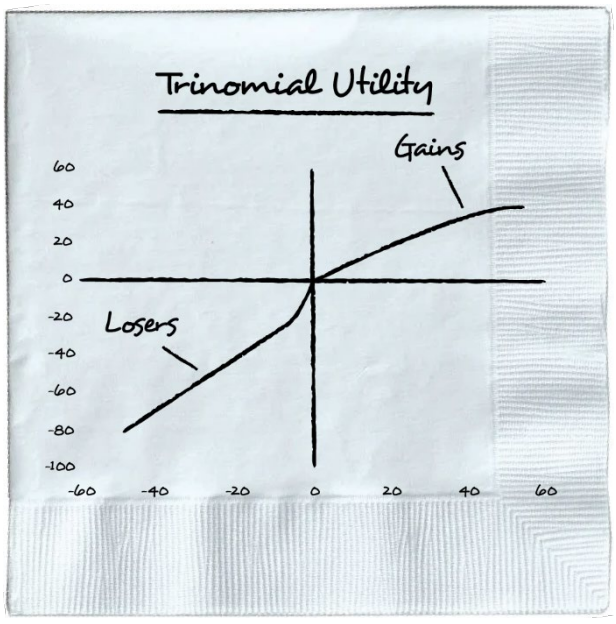


Napkin 4: Prospect Theory

Daniel Kahneman and Amos Tversky (1979) overhear this conversation and come barging over. They tell Locke that he needn't follow all of John and Oskar's rules. In fact, they think it is great that Locke is concerned about his prospects. Dick (Richard Thaler) joins in the conversation and emphasizes that it is fine with him if John Locke looks at each investment separately. Harry, at a table nearby, nods in agreement, "That's what I have been saying all along."

Dan, Amos and Dick tell Locke that it is OK to worry about losing the money he has. Even a small loss hurts, and it is going to take a bigger gain to offset that. They draw a picture of their utility function. They go on to tell Locke that he can even change the probabilities that he assigns to each outcome. Locke asks them to save that conversation for another day.

Locke sips his beer and thinks for a while. He's grateful for their help and feels that all of them have made good points. Locke wants to please everyone, but also wants to have a utility function that feels good to him. He writes down a trinomial function that takes a little from Nick and Daniel, John and Oskar, and Daniel, Amos, and Dick. None of them are happy with Locke.



Napkin 5: Trinomial Utility

Locke's utility function looks like this:

$$U(y) = y + a\,y^{b} + c\,(-y)^{d}$$

Where, $a < 0$, $b > 1$, where $c = 0$ if $y \dashrightarrow 0$, otherwise $c < 0$, $0 < d < 1$.

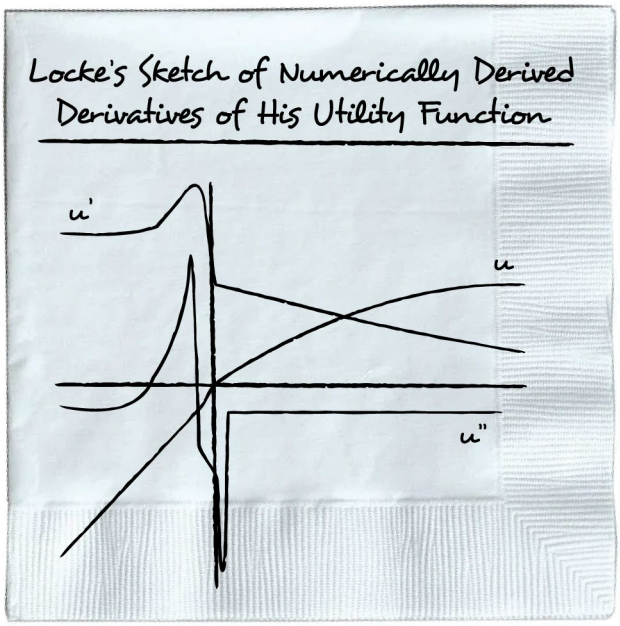
The first term in the utility function represents the Bernoulli expected value, the second term represents von Neuman Morgenstern's risk aversion and the third term only kicks in for negative values and reflects Kahneman-Tversky's prospect theory loss aversion. And, as suggested by Thaler and Markowitz, Locke is going to apply this function to every investment individually.

Locke sets $a = -0.25$, $b = 2$, $c = -0.31$ when $y < 0$ and sets $d = 0.25$, and draws a picture of his utility curve. Locke feels good about it.

All of a sudden, a nasty mathematics brawl breaks out. John and Oskar insist that Locke compute the first and second derivatives of his curve.

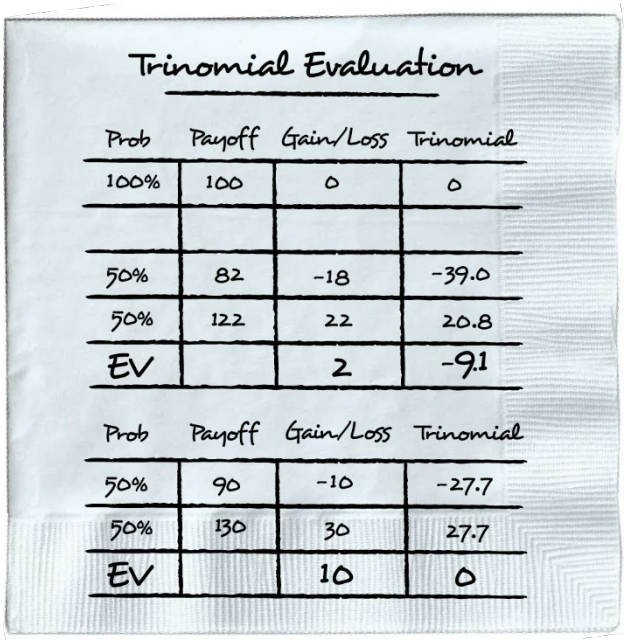
U'(y), the first derivative, is not continuous, there is a big jump at zero. Locke says, the fact that U' is not continuous poses issues in mathematics, but it poses no issues to his decision making. Payoffs from financial transactions are discrete so there is no risk of infinitesimal changes in payoffs creating infinite changes in his behavior. The mathematicians are not amused.

Moreover, U'(y) is positive below the bliss point [y = -(1/ab)^(1/b-1), for b =2, y = -1/2a], but then turns negative. To avoid escalating the fight, Locke agrees not to use it for very large risks.



Napkin 6: Locke's Sketch of Numerically Derived Derivates of His Utility Function

The second derivative is another problem for John and Oskar. Above zero it looks great, negative all the way— risk aversion. [U''(y) is negative for y greater than 0.] However, for small values of y below 0, U''(y) is positive. At more negative values of y U''(y) once again turns negative. [U''(y) is negative above 0 and for y < (b/8a)^(2/3)] John and Oskar are apoplectic, but Daniel, Amos and Dick assure him that it will all work out and offer to buy another round.



Napkin 7: Trinomial Evaluation

With his utility function now in hand, John Locke calculates the minimum return he will require to invest in Thomas's venture. John now looks at the gain and loss on the investment. If he gets 82 coins back that is a loss of 18 and if he gets 122 coins that is a gain of 22. When he plugs it into the equation, he has an expected loss of utility of 9.1 coins.

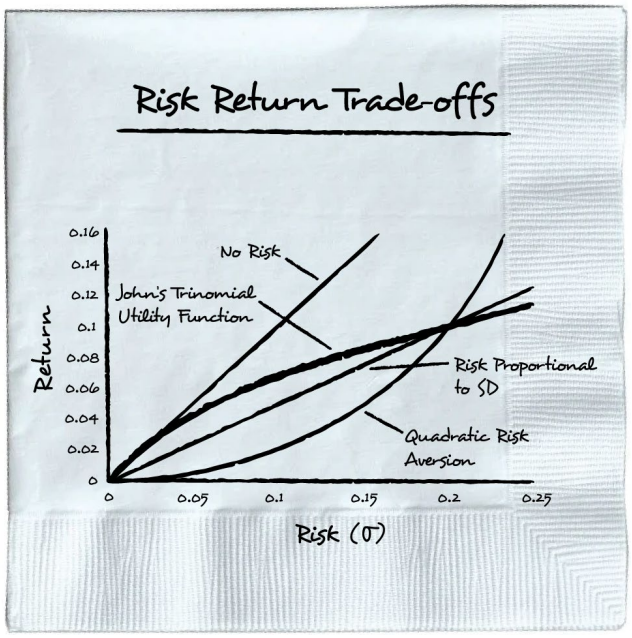
John says he will take the investment if Hobbs can offer him a 50% chance of 130 and a 50% chance of 90. Hobbes says this is highway robbery and starts to walk out of the pub.

Hobbes's choice

As Hobbes is storming out of The Whale, he stops to think. Hobbes does not have sufficient resources to create the venture himself, so he can only create the venture if the profits from the venture exceed John's required return. Hobbes thinks maybe he is being too rash, maybe he must agree to Locke's terms. After all, Locke has the money.

It becomes clear to Locke and Hobbes that the only ventures in the financial state of nature are ventures with returns above the investor's required rate of return. How high above the required return reflects the relative bargaining power of John and Thomas, but John will never accept a project below his required return, since he always has the option to remain in his risk-free assets (property or cash).

As Hobbes sulks back to the bar, he sees a group of people around a table talking about the benefits of a financial society with intermediaries. Thomas asks Locke to leave the utility theorists behind and join him at the table with financial experts who are recruiting people to leave the state of nature and join society.



Napkin 8: Risk Return Trade-offs

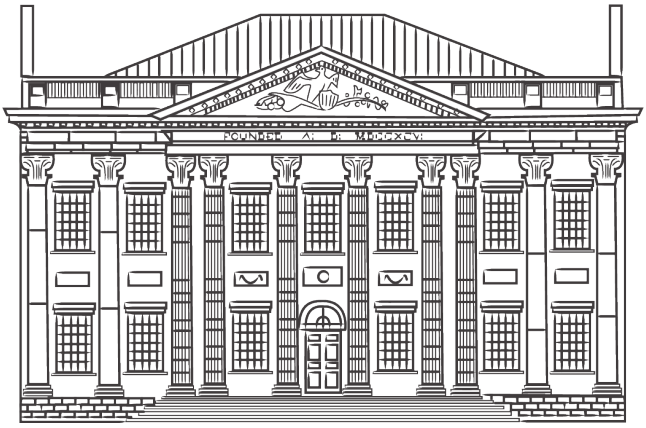
Locke isn't willing to go into a conversation with some slick financial types unarmed, so with the help of the utility theorists he builds a table that shows how much return he needs for ventures with different risk based on the risk of the venture and draws the results on a napkin. (See Segal & Spivak 1990). To get a better understanding of what he wants, John draws the risk-return relationships. The 45-degree line is the "no risk" line where the incremental return equals the amount of risk, and the other straight line is a lower level of risk compensation for proportional to the standard deviation of risk.

The upward curving line is quadratic risk aversion. Finally, the line in bold, curving downward, is derived from John's trinomial utility function. For low risk levels, John wants to be fully compensated for the risk, but at higher risk levels he doesn't need as much compensation. John realizes he likes both no risk and high risk, but in between doesn't do much for him. (See Davidson 2019 for an overview of data supporting this pricing function.) Armed with his charts, Locke is ready to face the finance gurus.

At the table with the intermediaries

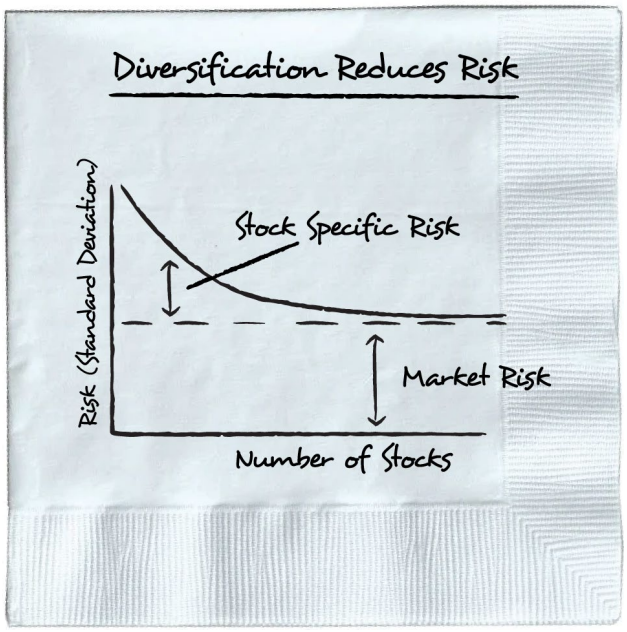
First, they meet Alexander Hamilton. He says that John can invest his money with the government and earn a guaranteed rate of return. Since the government can always print more money, there is no risk that he won't get his money back. John says this is a good idea, if the government agrees not to print too much money. Thomas is not thrilled with this idea since now he needs to compete with the government for John's money.

Alexander also says the government will be lending some money to banks and make sure that they have enough capital so John might want to lend to them. John is hesitant to deposit money in the banks, but if they build buildings that look like Greek temples he might be interested. Alexander shows John a postcard of the bank. Thomas also likes the idea of the government helping banks, because perhaps the banks will lend to him at a better rate than John.



First Bank of the United States (1795)

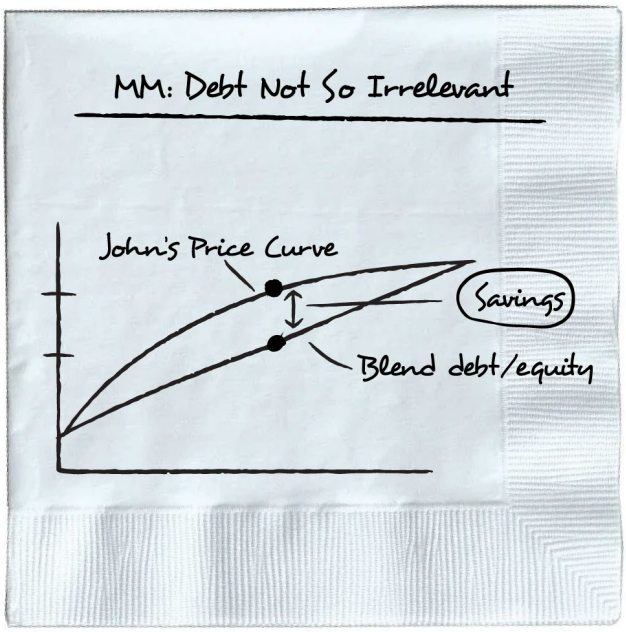
Harry, who you may recall was listening at the next table, ambles over and tells John that if he invested in a basket of companies, he would have a lot less risk (Markowitz 1952). John agrees; but says he can't afford to invest in so many companies. He doesn't have the time to sign contracts with everyone and then he will be stuck with a lot of small investments that no one else wants. Harry leans over and whispers to Thomas. Harry has an idea. He convinces Thomas that if he and other companies can make it easier for John and others to buy and sell shares of his company then Harry can set up a mutual fund (or ETF) that is a diversified portfolio. If that happens Harry can raise money at a much lower cost and pass on the savings to Thomas. They ask John what he thinks. John says, either way it's the same to me, I have my table and chart and as long as I get paid for my risk, I am indifferent to whether you are in a mutual fund or not. But John agrees that Thomas will have a lower required return if he can get his stock into a fund.



Napkin 9: Diversification Reduces Risk

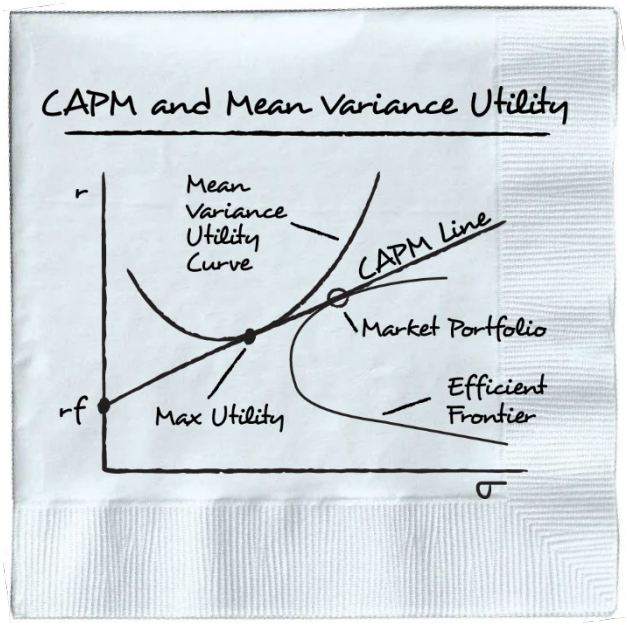
Thomas is now considering borrowing from the bank and registering his stock on an exchange but can't decide how much he should borrow. He talks to Franco and Merton (MM 1958). They say other than taxes and maybe a few other minor considerations it doesn't matter how

much debt you issue, it's all the same in the end. Thomas asks John what he thinks. John shows him his pricing curve. It's clear that John has a lower required return for a mix of debt and equity than for a low risk company. Franco and Merton tell John he is wrong since it is the same risk. John says he looks at each investment individually and isn't interested in their theories.



Napkin 10: MM Irrelevance of Debt

William (Sharpe 1964) comes along tells John that he can invest in both risk-free assets and Harry's mutual fund. He tells John that if he draws a line from the risk-free rate to the market rate, he can get rid of his own pricing model and just price to the market. William explains that the market line from the risk-free rate to the market portfolio is the mean variance maximizing return. However, when John draws the line, he sees that his required returns are above the line, and he wouldn't invest in the companies unless they provided a higher return. John says thanks, but no thanks to William. Your model doesn't make any sense. If I stay out of society I can earn more or stay with the risk-free asset. I don't believe in your capital markets line. Your analysis was for some abstract mean-variance investor, not for me.



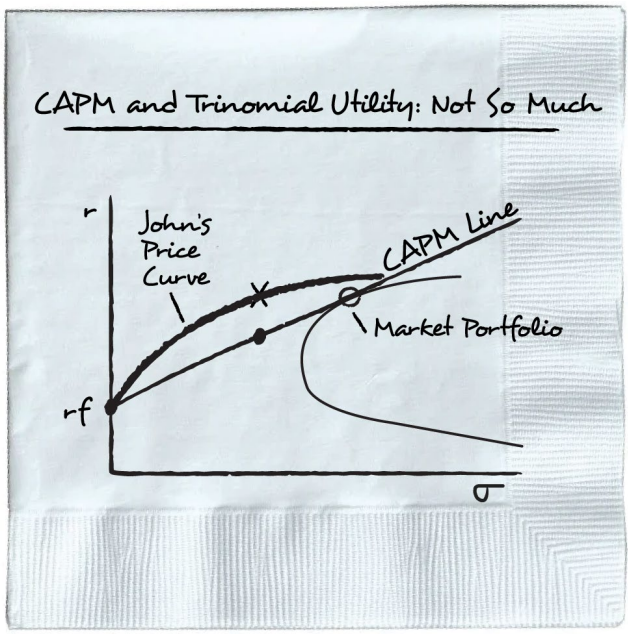
Napkin 11: CAPM and Mean Variance Utility

Fischer (Black 1972) says that he can draw the line to a higher point on the return axis at the zero-risk point and that is his implied cost of borrowing. John nods his head and says that line looks better, but I don't understand what you are talking about since I am not borrowing any money to buy Thomas's company.

On the other hand, Thomas likes the direction of the conversation. Whether it's the MM theorem or Sharpe's Model, it seems that Thomas gets a lower cost of funds if he joins society, borrows some money, and gets his stock into a mutual fund. Society also has a law: The Law of One Price. Thomas really likes this law; it says that it doesn't matter how he splits up the financing of his company. It's always the same value. No more need to negotiate with that highway robber Locke.

Maybe Locke has had a few too many, but he starts laughing. John says: "You keep your Theorems, Models and Laws. I have my own utility function and calculations. If you ask me none of these make sense. You keep saying that these portfolios maximize my utility, but I have my risk-return tradeoffs in my table and your analysis just doesn't do it for me."

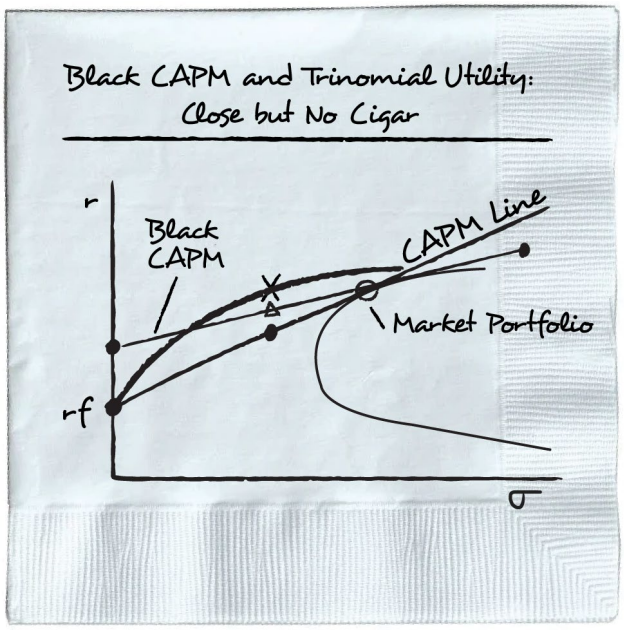
John starts to head for the door. Thomas pleads with John. He sees that the intermediaries can lower is his cost of funds. He asks the experts to plead with John and get him to join the society. Alexander, always the practical one, stops John by the door. "Look John," he says, "you don't need to believe these Theories, Models and Laws, all you need to do is make sure you get what you want."



Napkin 12: CAPM and Trinomial Utility: Not So Much

John is listening.

Alexander continues, "If you put your money in the bank, the government will make sure it's safe. You can be happy with the risk-free rate for that money."



Napkin 13: Black CAPM and Trinomial Utility: Close but No Cigar

"Go on."

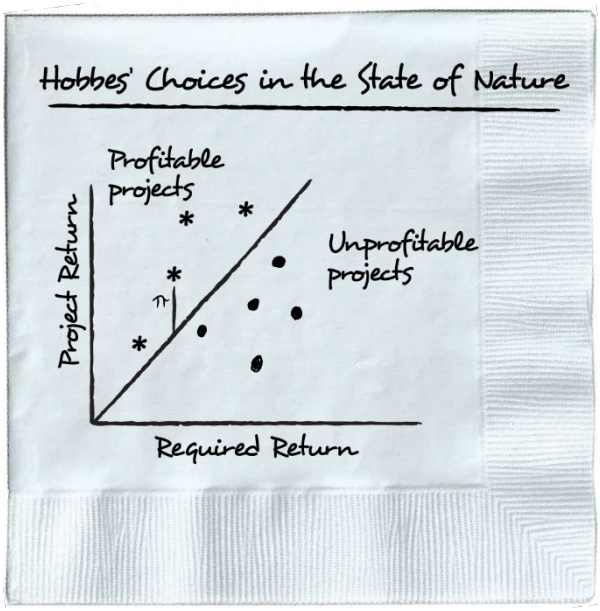
"And if you invest in Harry's mutual fund, you'll get paid for the undiversified risk. Right."

"But what about all those theories?" John asks.

"Those theories were made up to explain what people saw. But those theories require the intermediaries like the banks and the mutual funds to make them true. It's not your problem. You invest with your pricing function and the intermediaries will figure out how to get you what you want, the same as you would get in the financial state of nature. The net effect may be something that looks like CAPM and all that, but maybe not."

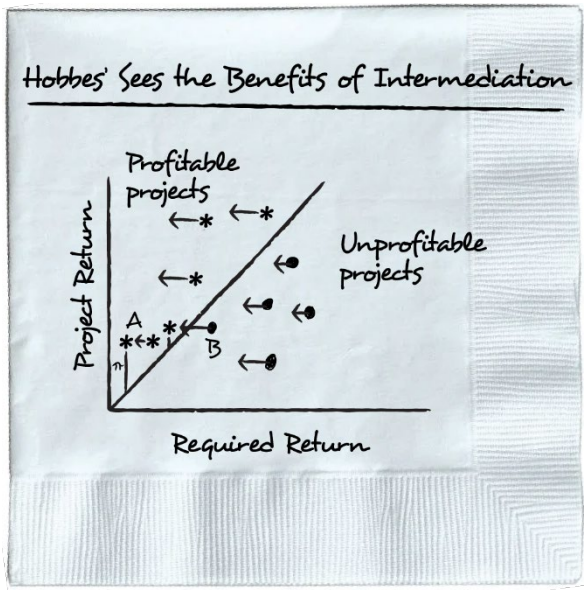
Thomas thinks he understands how intermediation works. He asks for a napkin and draws his understanding of how he can evaluate ventures. For each project he can plot the expected return of the project versus the required return for the project. He plots projects he is considering versus Locke's required returns in the State of Nature. Projects above the diagonal are profitable because the project return exceeds the required return.

With intermediation the required return of the projects decreases. Thomas draws small arrows from each project to the left to reflect the lower required return. Investments like Project A become more profitable and investments like Project B go from being unprofitable to being profitable.



Napkin 14: Hobbes' Choices in the State of Nature

John walks back to the table with the intermediaries. John realizes that he would be happy to have someone else make sure that Thomas doesn't lie to him and that he pays the money owed to him on time. He would also be happy to earn a return from the government or the bank rather than keeping gold coins under his bed with the fear that someone will steal them. He knows he would be making less money than investing with Thomas directly, but he is still happy with the risk-return tradeoff for his investments. He won't lose anything on his risk-free money and that he has diversified his risky investments so he can lower his return target.



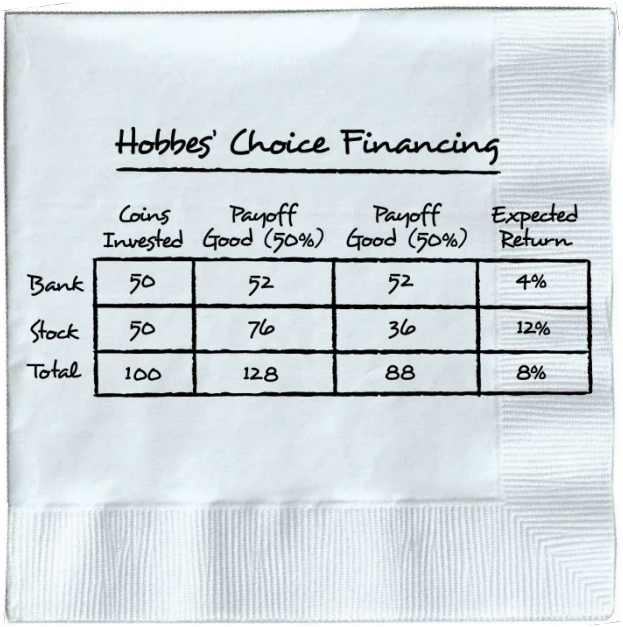
Napkin 15: Hobbes' Sees the Benefits of Intermediation

Thomas is even happier. Hobbes goes on to show everyone how if he can find a better execution for his financing with intermediaries; using intermediaries will lower the required return for each project. The intermediaries can provide for better execution than Locke using diversification, leverage, and the creation of risk-free assets. With better execution, his profits on ventures that were already profitable increases and that some new ventures that were not profitable, become profitable.

Thomas looks at the choices and determines his best execution to lower his financing cost.

He borrows 50% of the money at 4% from the bank and sells his stock into the market at 12%. His net cost is 8%. To get 100 gold coins he now only promises 88 when things go bad, and 128 when things go well. So now he is saving two gold coins versus the deal with Locke. Leaving the state of nature has made him better off. Hobbes pockets the two gold coins that would have gone to Locke and starts to dream of other ventures.

At another table, an argument breaks out. Adam has a lot of ideas on how the society can build wealth. Karl says he is all wrong. They keep debating while John and Thomas grab a few drinks at the bar and celebrate financial intermediation. Everyone is happy, even if they don't agree.



Napkin 16: Hobbes' Choice Financing

The story is a work of fiction that offers commentary on the various thinkers identified. Quotations and opinions are the author's own and not those of any of the characters real or imagined.

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